

Bartók, Fibonacci, and the Golden Ratio: Fact or Fiction

Gareth E. Roberts

Department of Mathematics and Computer Science
College of the Holy Cross
Worcester, MA, USA

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Béla Bartók

- **Béla Bartók:** Born in Nagyszentmiklós, Hungary (now Sînnicolau Mare, Romania) in 1881. Died in New York, Sept. 1945.
- Studied at the Catholic Gymnasium (high school) in Pozsony where he excelled in **math**, physics, and music. Enters the Academy of Music (Liszt was 1st president) in Budapest in 1899.
- Avid collector of folk music (particularly Hungarian, Romanian, Slovakian and Turkish).
- Influenced by Debussy and Ravel; preferred Bach to Beethoven.
- Considered to be one of Hungary's greatest composers.



Figure: Bartók at age 22.

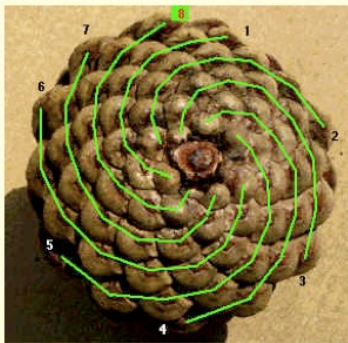
- Very interested in **nature**. Builds impressive collection of plants, insects, and minerals. Fond of sunflowers and fir-cones.
- “We follow nature in composition ... folk music is a phenomenon of nature. Its formations developed as spontaneously as other living natural organisms: the flowers, animals, etc.” — Bartók, *At the Sources of Folk Music* (1925)
- Notoriously silent about his own compositions. “Let my music speak for itself, I lay no claim to any explanation of my works!”



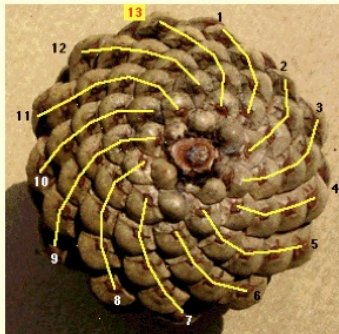
Figure: Red Columbine (left, 5 petals, source: ljhimages/iStock/Thinkstock); Black-eyed Susan (right, 13 petals, source: herreid/iStock/Thinkstock)



Figure: Chicory (left, 21 petals, source: ArminStautBerlin/iStock/Thinkstock); Sunflower (right, 34 petals, source: Racide/iStock/Thinkstock)



Adjacent
Fibonacci
numbers, 8, 13



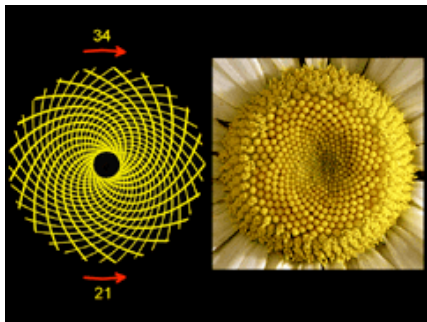


Figure: In most daisy or sunflower blossoms, the number of seeds in spirals of opposite direction are consecutive Fibonacci numbers.

Fibonacci Phyllotaxis: The abundance of Fibonacci examples in nature can be explained with [continued fractions](#) and the [golden ratio](#).

Plant Spirals: Beauty You Can Count On — exhibit at the Botanical Garden of Smith College (2002–03)

Ernő Lendvai

- In 1955, the Hungarian musical analyst **Ernő Lendvai** started to publish works claiming the existence of the **Fibonacci numbers** and the **Golden Ratio** in many of Bartók's pieces.
- Some find Lendvai's work fascinating and build from his initial ideas; others find errors in his analysis and begin to discredit him. Lendvai becomes a controversial figure in the study of Bartók's music.
- Lendvai draws connections between Bartók's love of nature and "organic" folk music, with his compositional traits. He takes a broad view, examining **form** (structure of pieces, where climaxes occur, phrasing, etc.) as well as **tonality** (modes and intervals), in discerning a substantial use of the Golden Ratio and the Fibonacci numbers.

Music for Strings, Percussion and Celesta (1936), Movement I

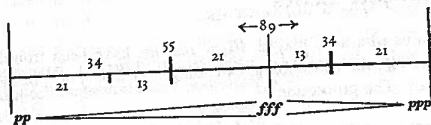


FIG. 22

Lendvai's analysis states:

- 1 Piece is 89 measures long.
- 2 The climax of the movement occurs at the end of bar 55 (loudest moment), which gives a subdivision of two Fibonacci numbers (34 and 55) that are an excellent approximation to the golden ratio.
- 3 String mutes are removed in measure 34.
- 4 The exposition in the opening ends after 21 bars.

Voilà: Fibonacci and the Golden Ratio!

Musik für Saiteninstrumente,
Schlagzeug und Celesta (in 4 Sätzen)

1

Musique pour instruments à cordes,
percussion et célesta (en 4 parties)

I.

Andante tranquillo, ♩ ca. 116-112

Béla Bartók

1.2. Viole *con sord.* *pp* 5

1.4. VI. *con sord.* *pp*

2. VIe. *con sord.* *pp*

2.4. VI. *con sord.* *pp*

5. VI. *con sord.* *pp* 10

1.4. VI. *con sord.* *pp*

2. VIe. *con sord.* *pp*

2. VIe. *con sord.* *pp*

2. VI. *con sord.* *pp* 15

1.4. VI. *con sord.* *pp*

2. VIe. *con sord.* *pp*

2. VIe. *con sord.* *pp*

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B. & H. 16155



Musical score for the first system, featuring the following instruments and parts:

- Cel.** (Celesta): A complex rhythmic pattern with sixteenth notes and sixteenth rests, marked with a '6' above the staff.
- 1. Vl.** (Violin I): A melodic line with a dotted line indicating a long note.
- 2. Vl.** (Violin II): A melodic line with a dotted line indicating a long note.
- 3. Vl.** (Violin III): A melodic line with a dotted line indicating a long note.
- 4. Vl.** (Violin IV): A melodic line with a dotted line indicating a long note.
- 1. Vle.** (Viola I): A melodic line with a dotted line indicating a long note.
- 2. Vle.** (Viola II): A melodic line with a dotted line indicating a long note.
- 1.2. Vlc.** (Violoncello I & II): A melodic line with a dotted line indicating a long note.

The second system includes:

- 1. Vl.** (Violin I): A melodic line with a dotted line indicating a long note.
- 2. Vl.** (Violin II): A melodic line with a dotted line indicating a long note.
- 3.4. Vl.** (Violin III & IV): A melodic line with a dotted line indicating a long note.
- 1.2. Vlc.** (Violoncello I & II): A melodic line with a dotted line indicating a long note.

The third system includes:

- 1. Vl.** (Violin I): A melodic line with a dotted line indicating a long note.
- 2. Vl.** (Violin II): A melodic line with a dotted line indicating a long note.
- 1.2. Vlc.** (Violoncello I & II): A melodic line with a dotted line indicating a long note.

Performance markings include *ppp* (pianissimo) and *poco rall.* (poco ritardando). A box containing the number 85 is present in the third system.

Problems with Lendvai's Analysis (Roy Howat)

- 1 The piece is 88 bars long, not 89! Lendvai includes a **footnote**:
“The 88 bars of the score must be completed by a whole-bar rest, in accordance with the Bülow analyses of Beethoven.” **Hanh?!**
- 2 The dynamic climax (fff) of the piece is certainly at the end of bar 55. But the tonal climax is really at bar 44, when the subject returns a tritone away from the opening A to E \flat . ($88/2 = 44$, symmetry?)
- 3 The viola mutes come off at the end of bar 33 (not 34) while the first violins and cellos remove their mutes at the start of measure 35 (again, not 34). Only the second, third, and fourth violins remove their mutes in bar 34.
- 4 The fugal exposition actually ends in bar 20, not 21.
- 5 What about the **celesta**? It enters after bar 77.

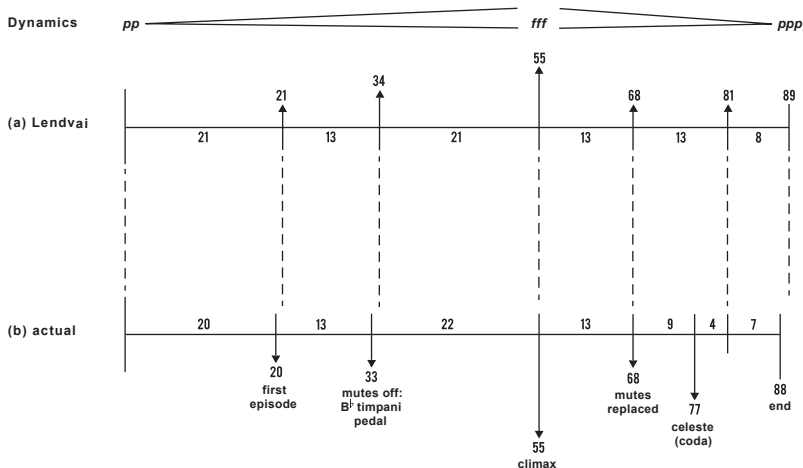
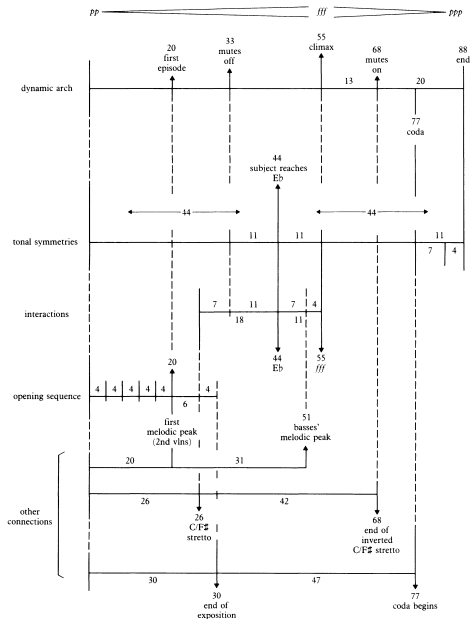


Figure: Roy Howat’s analysis of Lendvai’s work, from “Bartók, Lendvai and the Principles of Proportional Analysis,” *Music Analysis*, 2, No. 1 (March, 1983), pp. 69-95.

Fig. 5: Fugue from Music for Strings, Percussion and Celeste



A Magnificent Inversion

A dramatic and revealing **exact inversion** based on the scale of the main theme occurs at the end of the first movement.



- Top part is first violins; bottom part is second violins. All other instruments are silent.
- The inversion is about A, reaffirming it as the tonal center of the movement. The motion from A to E \flat and back to A recaps the tonal structure of the fugue. Key idea: **symmetry**.
- Who was the master of using inversions in fugues? **Bach!**
- Last four notes: C B \flat B \flat A, which translates in German (Bach's native tongue) to C H B A. **Coincidence?**

Music for Strings, Percussion and Celesta, Movement III

Adagio (♩ = 66)

mf 1 1 2 3 5 8 5 3 2 1 1

- Opening xylophone solo has the rhythmic pattern

1, 1, 2, 3, 5, 8, 5, 3, 2, 1, 1,

with a crescendo followed by a decrescendo ([hairpin](#)) climaxing at the top of the sequence. Obvious nod to **Fibonacci** as well as a nice use of retrograde symmetry.

- Music for this movement famously used by Stanley Kubrick in his film adaptation of Stephen King's *The Shining*.

Fig 7: third movement of *Music for Strings, Percussion and Celeste*

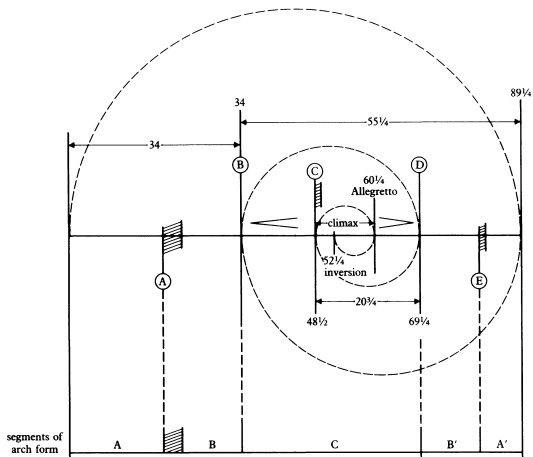


Figure: Howat's analysis of the third movement of *Music for Strings, Percussion and Celesta* from "Bartók, Lendvai and the Principles of Proportional Analysis," *Music Analysis*.

Some Final Remarks on the Bartók Controversy

- Lendvai's inaccuracies partly due to a narrow focus on the Fibonacci numbers. It's clear that the **Lucas numbers** were just as significant in the first movement of *Music for Strings, Percussion and Celesta*.
- Strength of first movement lies in its use of **symmetry**:
 - 1 Tonal climax in measure 44, half way through piece.
 - 2 Inverting the subject exactly after the climax in measure 55.
 - 3 Tonal symmetry built around A; mirrored trip around circle of fifths.
 - 4 Wonderful exact inversion at the end of the piece.

Final Remarks (cont.)

- Other works by Bartók where the golden ratio can be detected are *Sonata for Two Pianos and Percussion*, *Miraculous Mandarin*, and *Divertimento*.
- Bartók was highly secretive about his works. Surviving manuscripts of many of the pieces where the golden ratio appears to have been used contain no mention of it.
- Bartók was already being criticized for being too “cerebral” in his music. Identifying the mathematical patterns in structure and tonality (even to his students) would only have added fuel to the fire.
- **Bottom line:** Plenty of evidence in support of mathematical ideas at work in *Music for Strings, Percussion and Celesta*, but **don't fudge the analysis!**